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THE STABILITY OF THE STEADY MOTIONS OF A RIGID BODY IN A CENTRAL FIELD†

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The problem of the translational–rotational motion of a rigid body with a triaxial ellipsoid of inertia in a central gravitational field is considered. The body is modelled by a weightless sphere, at the ends of the three mutually perpendicular diameters of which there are point masses. It is shown that, unlike the cases when the approximate expression for the potential of the gravity forces is used, there are not only "trivial" steady motions of the body, for which the main central axes of inertia of the body coincide with the axes of the orbital system of coordinates, but also other classes of steady motions. In addition, the stability of these "trivial" steady motions is investigated, and the possibility of secular stability of the motions, unstable in the satellite approximation, is pointed out.

1. We will consider the motion of a body with a triaxial ellipsoid of inertia in a central Newtonian field. The body is modelled by a weightless sphere of radius a, at the ends of the three mutually orthogonal diameters d_i of which there are point masses $m_i/2$, $i = 1, 2, 3$. Without loss of generality we will assume that $m_1 > m_2 > m_3$.

We will introduce a fixed system of coordinates $M\xi\eta\zeta$ with origin at the attracting centre and system of coordinates $Ox_1x_2x_3$ connected to the body with origin at the centre of mass and axes Ox_i directed along the above-mentioned diameters. The position of the centre of mass of the body with respect to the fixed system of coordinates will be defined by the spherical coordinates ρ, ϑ, ψ, where ϑ is the angle that the radius vector $\rho = MO$ makes with the plane $O\zeta\xi$, ψ is the angle between $O\zeta$ axis and the projection of the radius vector ρ on the $O\zeta\xi$ plane, and ρ is the length of the vector ρ.

Suppose γ is the unit vector directed along the vector ρ, while β is the unit vector directed along the $M\eta$ axis. The projections of these vectors onto the Ox_i axes will be denoted by γ_i and β_i, respectively. They are obviously linked by the relations

$$\sum \gamma_i^2 = 1, \ \sum \beta_i^2 = 1, \ \sum \gamma_i\beta_i = \sin\vartheta \tag{1.1}$$

Suppose $m = m_1 + m_2 + m_3$ is the mass of the body, $J_i = a^2(m_j + m_k)$ are its principal central moments of inertia, and $i \neq j \neq k$, $(i, j, k) \in S_3$, where S_3 is the group of permutations of the three elements (1, 2, 3); here $J_1 < J_2 < J_3$.

The kinetic and potential energies of the system have the form

$$T = \frac{1}{2}[m(\dot\rho^2 + \rho^2\dot\psi^2\cos^2\vartheta + \rho^2\dot\vartheta^2) + J_1\omega_1^2 + J_2\omega_2^2 + J_3\omega_3^2]$$

$$U = \sum \ [F_i(a) + F_i(-a)], \ \ F_i(a) = -\frac{fMm_i}{2}(\rho^2 + a^2 + 2a\rho\gamma_i)^{-1/2}$$

Here f is the gravitational constant, M is the mass of the attracting centre, and ω_i are the projections of the absolute angular velocity of the body ω onto the Ox_i axes.

The equations of motion of the body allow of energy and area integrals

$$T + U = \text{const}, \ \ \partial T / \partial\dot\psi = k = \text{const}$$

Assuming $\omega = \dot\psi\beta + \omega^*$, where ω^* is the instantaneous angular velocity of the body in its motion in an orbital system of coordinates and ignoring the cyclic coordinate ψ, we will introduce the Routh function

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$$R = T - U - k\psi \equiv R_2 + R_1 + R_0$$

Here R_s is the homogeneous part of the function R of degree s with respect to $\dot\rho$, $\dot\vartheta$, ω_i^*. Here, the change potential energy of the body $W = -R_0$ takes the form $W = U + k^2/(2S)$, where

$$S = m\rho^2\cos^2\vartheta + J_1\beta_1^2 + J_2\beta_2^2 + J_3\beta_3^2$$

By Routh's theorem the steady motions of the system are denoted by the critical points of the function W (see, for example, [1–3]).

Since the direction cosines are linked by relations (1.1), instead of W we will henceforth consider the function

$$W_* = W + \lambda(\sum \gamma_i\beta_i - \sin\vartheta) + \sigma(\sum \gamma_i^2 - 1)/2 + \nu(\sum \beta_i^2 - 1)/2$$

where λ, σ, ν are undetermined Lagrange multipliers.

2. The conditions for the function W_* to be stationary lead to the following equations

$$\frac{\partial W_\star}{\partial\rho} = \frac{fM}{2}\sum m_i\big((\rho + a\gamma_i)P_i(a) + (\rho - a\gamma_i)P_i(-a)\big) - \frac{k^2 m\rho\cos^2\vartheta}{S^2} = 0 \tag{2.1}$$

$$\frac{\partial W_\star}{\partial\vartheta} = \frac{k^2 m\rho\sin 2\vartheta}{2S^2} - \lambda\cos\vartheta = 0 \tag{2.2}$$

$$\frac{\partial W_\star}{\partial\gamma_i} = \frac{afM\rho}{2} m_i(P_i(a) - P_i(-a)) + \lambda\beta_i + \sigma\gamma_i = 0 \tag{2.3}$$

$$\frac{\partial W_\star}{\partial\beta_i} = -\frac{k^2}{2S^2}J_i\beta_i + \lambda\gamma_i + \nu\beta_i = 0 \tag{2.4}$$

Here $Pi(a) = (\rho^2 + a^2 + 2a\rho\gamma_i)^{-3/2}$.

Solutions of the form

$$\gamma_i^2 = 1,\ \beta_j^2 = 1\ (i \neq j),\ \gamma_j = \gamma_k = \beta_i = \beta_k = 0$$

$$\vartheta = 0,\ \lambda = 0,\ \sigma = \sigma_{0i},\ \nu = \nu_{0j} \tag{2.5}$$

obviously satisfy system (2.2)–(2.4), (1.1) identically with respect to ρ.

The solutions correspond to steady motions of the body for which its centre of mass moves in a circular orbit, one of the principal central axes of inertia of which is directed along the radius vector, while the other two are directed along the tangent and binormal to the orbit. Equation (2.1) then takes the form

$$k^2 = Cy_{ij}(\rho) \tag{2.6}$$

$$C = \frac{fMS^2}{m\rho},\quad y_{ij}(\rho) = m_i\frac{\rho^2 + a^2}{\rho(\rho^2 - a^2)^2} + \frac{m_j + m_k}{(\rho^2 + a^2)^{3/2}}$$

the constants σ_{0i} and ν_{0i} are defined by the relations

$$\sigma_{0i} = fMm_i a^2\frac{\rho(3\rho^2 + a^2)}{(\rho^2 - a^2)^3},\quad \nu_{0j} = \frac{k^2 J_j}{S^2} \tag{2.7}$$

When $\rho > a$ there is a unique point ρ_{ij}^0 such that $y_{ij}'(\rho_{ij}^0) = 0$. Consequently, Eq. (2.6), when $\rho > a$ has no solutions for $k^2 < k_{ij}^{02} = y_{ij}(\rho_{ij}^0)$, has a unique solution $\rho = \rho_{ij}^0$ for $k^2 = k_{ij}^{02}$ and two solutions $\rho = \rho_{ij}^{\pm}(k^2)$ for $k^2 > k_{ij}^{02}$, where $\rho_{ij}^+ > \rho_{ij}^0 > \rho_{ij}^-$ and $y_{ij}' > 0$ when $\rho > \rho_{ij}^0$, $y_{ij}' < 0$ when $\rho < \rho_{ij}^0$.

In other words, for a specified value of the constant of the area integral two different steady motions of the body are possible corresponding to the same orientation of the body and differing in the value of the radius of the orbit (compare with [4–7]).

Henceforth, without loss of generality we will consider the solutions (2.8) for which $\gamma_i = \beta_j = 1$, since the solutions of the form $\gamma_i \pm 1$, $\beta_j = \pm 1$ correspond to geometrically identical solutions.

3. To investigate the stability of the steady motions we set up the secular equation [3]

$$\Delta(\kappa) = (p_1 - \kappa)(p_2 - \kappa)(p_3 - \kappa)(3\kappa^2 - 2\kappa p_4 + p_5) = 0$$

where p_s are expressed in a certain way (see below) in terms of the diagonal elements of the matrix of the second variation of the reduced potential ($n = 1, 2, 3$)

$$w_{11} = \left(\frac{\partial^2 W_*}{\partial \rho^2}\right)_0, \quad w_{22} = \left(\frac{\partial^2 W_*}{\partial \vartheta^2}\right)_0 = \frac{k^2 m\rho^2}{S^2}$$

$$w^i_{n+2,n+2} = \left(\frac{\partial^2 W_*}{\partial \gamma_n^2}\right)_0 = fMa^2\rho\left[m_i \frac{(3\rho^2 + a^2)}{(\rho^2 - a^2)^3} - \frac{3m_n\rho}{(\rho^2 + a^2)^{5/2}}\right]$$

$$w_{n+5,n+5} = \left(\frac{\partial^2 W_*}{\partial \beta_n^2}\right)_0 = \frac{k^2 I_n}{S^2}, \quad \begin{array}{l} I_1 = J_3 - J_1, \quad I_2 = J_3 - J_2 \\ I_3 = J_2 - J_1 \end{array}$$

Here and henceforth the subscript zero denotes that the corresponding expression is calculated for the steady motion considered.

Consider the following steady motion

$$\rho = \rho_{1,3}(k^2), \quad \gamma_1 = \beta_3 = 1, \quad \gamma_2 = \gamma_3 = \beta_1 = \beta_2 = 0$$

$$\sigma = \sigma_{01}, \quad \nu = \nu_{03} \tag{3.1}$$

corresponding to the orientation of the body for which the axis of the least moment of inertia is directed along the radius vector, the axis of the greatest moment of inertia is directed along the normal to the plane of the orbit, and the axis of the mean moment of inertia is directed along the tangent.

Here and henceforth $\rho_{ij}(k^2)$ is the solution of Eq. (2.6), σ_{0i}, ν_{0j} are defined by (2.7) for appropriate values of i and j, while the relations $\vartheta = \lambda = 0$ are common for all the solutions of the form (2.5).

For the steady motion (3.1) the coefficients of the secular equation take the following form

$$p_1 = w_{11}, \quad p_2 = w^1_{44}, \quad p_3 = w_{77}$$

$$p_4 = w_{22} + w^1_{55} + w_{66}, \quad p_5 = w_{22}w^1_{55} + w_{22}w_{66} + w^1_{55}w_{66}$$

The sign of w_{11} is the same as the sign of $y'_{13}(\rho)$, i.e. $w_{11} > 0$ for $\rho = \rho^+_{13}(k^2)$, $w_{11} = 0$ for $\rho = \rho^0_{13}$ and $w_{11} < 0$ for $\rho = \rho^-_{13}(k^2)$, while $w_{22} > 0$, $w_{66} > 0$, $w_{77} > 0$ in view of the fact that $J_3 > J_2 > J_1$.

To investigate w^1_{44} and w^1_{55} we will consider the function $g_{ij}(x) = m_i^2 m_j^{-2} 9^{-1} x^{-1}(3x + b)^2 \cdot (x + b)^5 (x - b)^{-6}$ ($x = \rho^2$, $a = b^2$). It is obvious that

$$\lim_{x \to b+} g_{ij}(x) = +\infty, \quad \lim_{x \to +\infty} g_{ij}(x) = m_i^2 m_j^{-2}$$

where $g'_{ij}(x) < 0$ for any $x > b$. Hence, we obtain that $g_{ij}(x) > 1$ when $m_i > m_j$ and $g_{ij}(x) < 1$ otherwise.

The function $g_{12} > 1$. Consequently $w^1_{44} > 0$ and $w^1_{55} > 0$, and also (taking the relations $w_{22} > 0$, $w_{66} > 0$ into account) we have $p_4 > 0$ and $p_5 > 0$.

Hence, the four roots of the secular equation corresponding to solution (3.1) are always positive, while the root $\kappa_1 = w_{11}$ is positive for the branch $\rho = \rho^+_{13}(k^2)$ and negative for the branch $\rho = \rho^-_{13}(k^2)$

Thus, the degree of instability of solutions (3.1) is equal to zero for $\rho = \rho^+_{13}(k^2)$ and equal to unity for $\rho = \rho^-_{13}(k^2)$.

Consequently, the steady motions (3.1) are stable in the secular sense if $\rho = \rho^+_{13}(k^2)$ and unstable if $\rho = \rho^-_{13}(k^2)$.

For steady motion

$$\rho = \rho_{12}(k^2), \quad \gamma_1 = \beta_2 = 1, \quad \gamma_2 = \gamma_3 = \beta_1 = \beta_3 = 0, \quad \sigma = \sigma_{01}, \quad \nu = \nu_{02} \tag{3.2}$$

corresponding to the orientation of the body in which the axis of the least moment of inertia is directed along the radius vector, the axis of the mean moment of inertia is directed along the normal to the plane of the orbit and the axis of the greatest moment of inertia is directed along the tangent, we similarly obtain that the degree of instability of solutions (3.2) is equal to unity for $\rho = \rho_{12}^{+}(k^2)$ and equal to two for $\rho = \rho_{12}^{-}(k^2)$ (for these solutions one of the roots of the secular equation is always less than zero, three others are always greater than zero, and one changes sign when $\rho = \rho_{12}^{0}$).

Consequently, the steady motions (3.2) are unstable if $\rho = \rho_{12}^{+}(k^2)$, while the steady motions $\rho = \rho_{12}^{-}(k^2)$, generally speaking, gyroscopic stabilization is possible.

In Fig. 1 we show the form of the section of space ρ, ϑ, γ, β, k by the hyperplane $\gamma_1 = 1$, $\beta_{3(2)} = 1$, $\gamma_2 = \gamma_3 = 0$, $\beta_1 = \beta_{2(3)} = 0$, $\vartheta = 0$; the numbers 0(1) and 1(2) indicate the degree of instability of the corresponding steady motions.

4. Consider the steady motion

$$\rho = \rho_{23}(k^2),\ \gamma_2 = \beta_3 = 1,\ \gamma_1 = \gamma_3 = \beta_1 = \beta_2 = 0,\ \sigma = \sigma_{02},\ \nu = \nu_{03} \tag{4.1}$$

corresponding to the orientation of the body in which the axis of the least moment of inertia is directed along the tangent to the orbit, the axis of the mean moment of inertia is directed along the radius vector, and the axis of the greatest moment of inertia is directed along the normal to the plane of the orbit.

For steady motion (4.1) the coefficients of the secular equation take the following form

$$p_1 = w_{11},\ p_2 = w_{33}^2,\ p_3 = w_{66},\ p_4 = w_{22} + w_{55}^2 + w_{77},\ p_5 = w_{22}w_{55}^2 + w_{22}w_{77} + w_{55}^2 w_{77}$$

The sign of w_{11} is the same as the sign of $y_{23}'(\rho)$, i.e. $w_{11} > 0$ if $\rho = \rho_{23}^{+}(k^2)$, $w_{11} = 0$ if $\rho = \rho_{23}^{0}$ and $w_{11} < 0$ if $\rho = \rho_{23}^{-}(k^2)$. Obviously, $w_{22} > 0$, $w_{55} > 0$, since $m_2 > m_3$, while $w_{66} > 0$, $w_{77} > 0$ since $J_3 > J_2 > J_1$.

The function $g_{21}(x) = 1$ when $x = x_{23}^{*}$. Consequently, $w_{33}^2 > 0$ if $\rho \in (a;\ \rho_{23}^{*} = (x_{23}^{*})^{1/2})$, $w_{33} = 0$ if $\rho = \rho_{23}^{*}$ and $w_{33}^2 < 0$ if $\rho \in (\rho_{23}^{*};\ +\infty)$, $p_4 > 0$ and $p_5 > 0$. By what was said above the root $\kappa_1 = w_{11}$ is positive for the branch $\rho = \rho_{23}^{+}(k^2)$ and negative for the branch $\rho = \rho_{23}^{-}(k^2)$, $\kappa_2 > 0$ for $\rho \in (a,\ \rho_{23}^{*})$, $\kappa_2 = 0$ for $\rho = \rho_{23}^{*}$ and $\kappa_2 < 0$ for $\rho \in (\rho_{23}^{*};\ +\infty)$, while the remaining three roots are always positive.

Depending on the parameters of the problem $\mu_1 = m_1 m_3^{-1}$, $\mu_2 = m_2 m_3^{-1}$, two versions of the position of the points ρ_{23}^{0}, ρ_{23}^{*} are possible: (a) $\rho_{23}^{0}, > \rho_{23}^{*}$, (b) $\rho_{23}^{0}, > \rho_{23}^{*}$.

Consequently, the steady motions (4.1) in case (a) are unstable if $\rho \in (a;\ \rho_{23}^{*}) \cup (\rho_{23}^{0};\ +\infty)$, and unstable in the secular sense if $\rho \in (\rho_{23}^{*};\ \rho_{23}^{0})$, while in case (b) they are unstable if $\rho \in (a;\ \rho_{23}^{0}) \cup (\rho_{23}^{*};\ +\infty)$ and stable in the secular sense if $\rho \in (\rho_{23}^{0},\ \rho_{23}^{*})$, where in this case $\rho_{23}^{*} \to +\infty$ is $\mu_1 \to \mu_2$.

For the steady motion

$$\rho = \rho_{21}(k^2),\ \gamma_2 = \beta_1 = 1,\ \gamma_1 = \gamma_3 = \beta_2 = \beta_3 = 0,\ \sigma = \sigma_{02},\ \nu = \nu_{01} \tag{4.2}$$

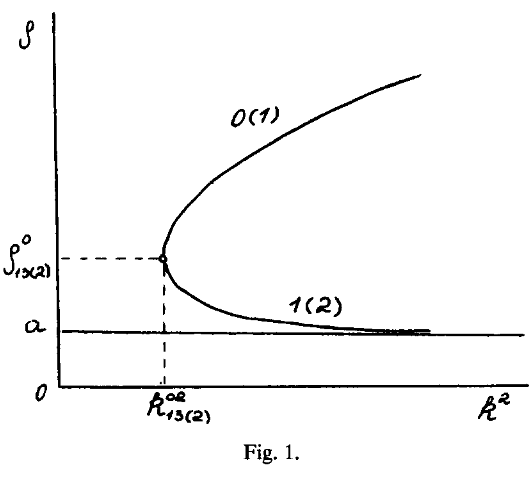

Fig. 1.

corresponding to the orientation of the body in which the axis of the mean moment of inertia is directed along the radius vector, the axis of the least moment of inertia is directed along the normal to the plane of the orbit, and the axis of the greatest moment of inertia is directed along the tangent, we similarly obtain that, depending on the parameters μ_1 and μ_2, the steady motions (4.2) in case (a) are unstable if $\rho \in (\rho_{21}^*, \rho_{21}^0)$ and unstable in the secular sense if $\rho \in (a; \rho_{21}^*) \cup (\rho_{21}^0, +\infty)$, while in case (b) they are unstable if $\rho \in (\rho_{21}^0, \rho_{21}^*)$ and unstable in the secular sense if $\rho \in (a, \rho_{21}^0) \cup (\rho_{21}^*, +\infty)$.

In Fig. 2 we show the form of the section of the space ρ, ϑ, γ, β, k by the hyperplane $\gamma_2 = 1$, $\beta_{3(1)} = 1, \gamma_1 = \gamma_3 = 0, \beta_2 = \beta_{1(3)} = 0, \vartheta = 0$, where the numbers 0(1), 1(2) and 2(3) indicate the degree of instability of the corresponding steady motions. Note that at the points $S_{23(1)}$ the degree of instability of the corresponding motions changes despite the fact that at these points, it would appear, there is no branching of the solutions. In fact, at these points the expression $w_{33}^2(p_{21})$ vanishes and from solutions (4.1) and (4.2) the steady motions for which

$$\gamma_1^2 + (\gamma_2 - 1)^2 + \gamma_2^2 + \beta_{1(3)}^2 + \beta_2^2 + (\beta_{3(1)} - 1)^2 + \vartheta^2 \neq 0$$

branch off. The branches corresponding to these solutions emerge from the sections indicated in Fig. 2(a), (b).

The orientations of the body for which the axis of the greatest moment of inertia is directed along the normal to the plane of the orbit while the axis of the mean and least moments of inertia do not coincide with the radius vector of the centre of mass and the tangent to the orbit correspond to these solutions.

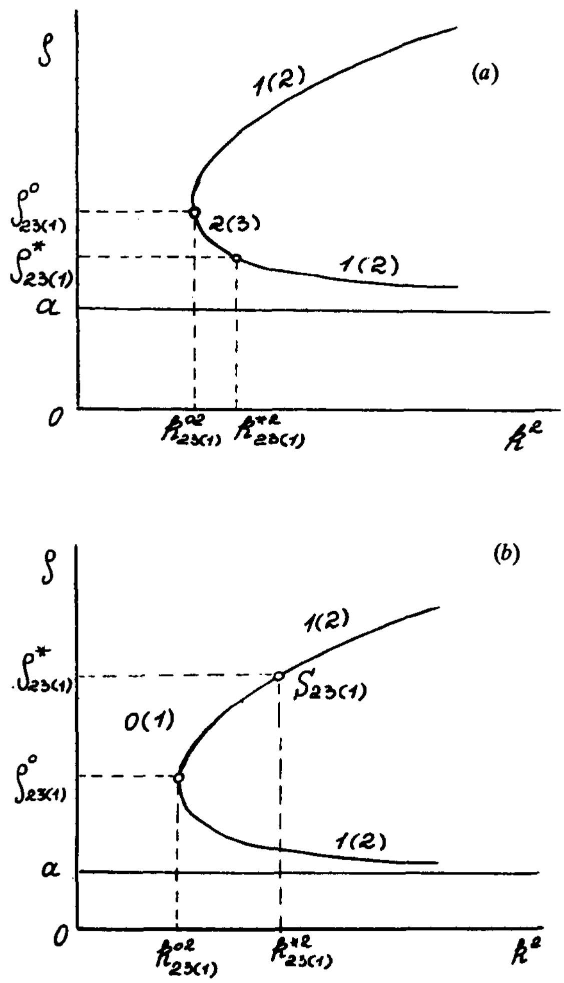

Fig. 2.

Note. If $\mu_1 \to \mu_2 \to +0$ then $\rho_{23}^* \to +\infty$. This means that secular stability of the steady motions of a body with a triaxial ellipsoid of inertia, close to an ellipsoid of revolution is possible if the axis of least moment of inertia is directed along the tangent to the orbit, the axis of the mean moment of inertia is directed along the radius vector, while the axis of the greatest moment of inertia is directed along the normal to the plane of the orbit (it is assumed here that the least and mean moments of inertia are close but not equal to one another (see also [4])).

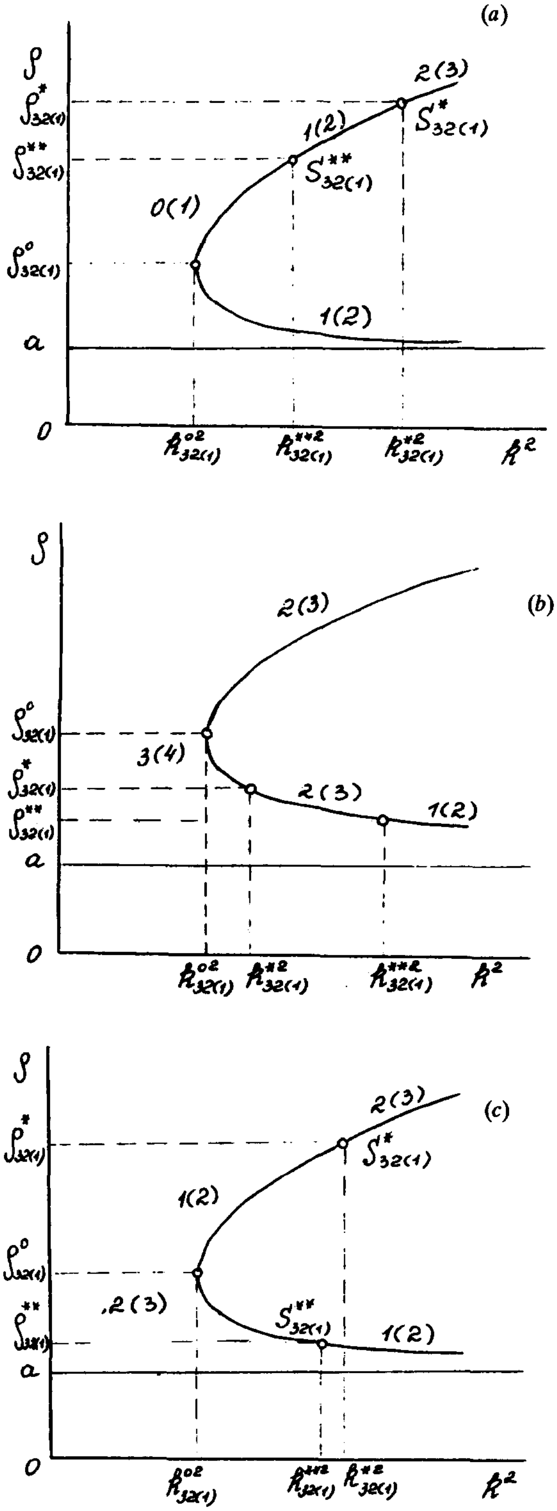

Fig. 3.

5. In a similar way we can investigate the stability of the steady motions

$$\rho=\rho_{32}(k^2),\ \gamma_3=\beta_2=1,\ \gamma_1=\gamma_2=\beta_1=\beta_3=0,\ \sigma=\sigma_{03},\ \nu=\nu_{02}, \tag{5.1}$$

$$\rho=\rho_{31}(k^2),\ \gamma_3=\beta_1=1,\ \gamma_1=\gamma_2=\beta_2=\beta_3=0,\ \sigma=\sigma_{03},\ \nu=\nu_{01} \tag{5.2}$$

which correspond to orientations of the body for which the axis of the greatest moment of inertia is directed along the radius vector, the axis of the mean moment of inertia is directed along the normal (5.1) or along the tangent (5.2), and the axis of least moment of inertia is directed along the tangent (5.1) or along the normal (5.2).

In Fig. 3(a)–(c) we show the form of the section of the space ρ, ϑ, γ, β, k by the hyperplane $\gamma_2 = 1$, $\beta_{3(1)} = 1, \gamma_1 = \gamma_3 = 0, \beta_2 = \beta_{1(3)} = 0, \vartheta = 0$ and we also show the degree of instability of the corresponding steady motions.

At the points $S^{*}_{32(1)}$ and $S^{**}_{32(1)}$ the degree of instability of the corresponding motions changes and the steady motions for which

$$\gamma_1^2+(\gamma_3-1)^2+\gamma_2^2+\beta_{1(2)}^2+\beta_3^2+(\beta_{2(1)}-1)^2+\vartheta^2\neq 0$$

are branched off from the solutions (5.1) and (5.2). The branches corresponding to these solutions emerge from the sections shown in Fig. 3(a)–(c).

Orientations of the body for which the axis of the mean (least) moment of inertia directed along the normal to the plane of the orbit while the axis of the greatest and least (mean) moments of inertia do not coincide with the radius vector of the centre of mass and the tangent to the orbit correspond to these solutions.

6. In conclusion we will compare the results obtained with the results of an investigation of the "satellite approximation" for the potential of the gravity forces. In the latter case it is assumed that $\varepsilon = a/\rho \ll 1$, and we must confine ourselves to that part of Figs 1–3 for which this assumption is satisfied. Then, for solutions of the form (3.1) and (3.2) the results obtained in Section 3 agree with the corresponding results for the case when the "satellite approximation" is used (see, for example, [1]).

For the solutions (4.1), (4.2) and (5.1), (5.2) in the general case there is no exact agreement. It occurs only when the quantity

$$\delta=\min_{(i,j,k)\in S_3}((m_i-m_j)/m_k)$$

differs from zero by a certain finite number (see Fig. 2a and Fig. 3b). If $\delta \ll 1$, the points S_{23} and S_{32} on the branches indicated by the plus sign (see Fig. 2b and Fig. 3a and c), may depart to infinity and the relation $\varepsilon \ll 1$ can be satisfied for them. Hence, the "satellite approximation" does not enable one, in particular, to detect the possibility that "nontrivial" steady motions will exist and the presence of secular stability of the "trivial" steady motions for which the axis of the least moment of inertia of the body is not directed along the radius vector, even when the assumption that the dimensions of the body are small compared with the radius of the orbit of its centre of mass is satisfied, if in this case the ratio of any two moments of inertia of the body is close to unity (see also [4, 8]). Note that the latter certainly occurs for many natural celestial bodies.

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